12. $x''_{tt} = a(ty'_t - y), \quad y''_{tt} = b(tx'_t - x).$

The transformation

$$u = tx_t - x, \qquad v = ty'_t - y \tag{1}$$

leads to the first-order system

$$u'_t = atv, \qquad v'_t = btu.$$

The general solution of this system is given by

$$\begin{aligned}
&\text{with } ab > 0: \quad \begin{cases} u(t) = C_1 a \exp\bigl(\tfrac{1}{2}\sqrt{ab}\,t^2\bigr) + C_2 a \exp\bigl(-\tfrac{1}{2}\sqrt{ab}\,t^2\bigr), \\ v(t) = C_1\sqrt{ab}\,\exp\bigl(\tfrac{1}{2}\sqrt{ab}\,t^2\bigr) - C_2\sqrt{ab}\,\exp\bigl(-\tfrac{1}{2}\sqrt{ab}\,t^2\bigr), \end{cases}\\
&\text{with } ab < 0: \quad \begin{cases} u(t) = C_1 a \cos\bigl(\tfrac{1}{2}\sqrt{|ab|}\,t^2\bigr) + C_2 a \sin\bigl(\tfrac{1}{2}\sqrt{|ab|}\,t^2\bigr), \\ v(t) = -C_1\sqrt{|ab|}\,\sin\bigl(\tfrac{1}{2}\sqrt{|ab|}\,t^2\bigr) + C_2\sqrt{|ab|}\,\cos\bigl(\tfrac{1}{2}\sqrt{|ab|}\,t^2\bigr), \end{cases}
\end{aligned} \tag{2}$$

where C_1 and C_2 are arbitrary constants. On substituting (2) into (1) and integrating the resulting expressions, one arrives at the general solution of the original system:

$$x = C_3 t + t\int \frac{u(t)}{t^2}\,dt, \qquad y = C_4 t + t\int \frac{v(t)}{t^2}\,dt,$$

where C_3 and C_4 are arbitrary constants.