



Systems of Ordinary Differential Equations > Linear Systems of Two Equations

14. $x''_{tt} = f(t)(a_1x'_t + b_1y'_t), \quad y''_{tt} = f(t)(a_2x'_t + b_2y'_t).$

Let k_1 and k_2 are roots of the quadratic equation

$$k^2 - (a_1 + b_2)k + a_1b_2 - a_2b_1 = 0.$$

Then the system in question can be reduced, by adding together the two equations multiplied by appropriate constants, to the following two independent equations:

$$\begin{aligned} z''_1 &= k_1 f(t) z'_1, & z_1 &= a_2 x + (k_1 - a_1) y; \\ z''_2 &= k_2 f(t) z'_2, & z_2 &= a_2 x + (k_2 - a_1) y. \end{aligned}$$

Integrating these and returning to the original variables, one arrives at a linear algebraic system for the unknowns x and y :

$$\begin{aligned} a_2 x + (k_1 - a_1) y &= C_1 \int \exp[k_1 F(t)] dt + C_2, \\ a_2 x + (k_2 - a_1) y &= C_3 \int \exp[k_2 F(t)] dt + C_4, \end{aligned}$$

where C_1, \dots, C_4 are arbitrary constants and $F(t) = \int f(t) dt$.