15. $x''_{tt} = af(t)(ty'_t - y), \quad y''_{tt} = bf(t)(tx'_t - x).$

The transformation

$$u = tx_t - x, \qquad v = ty'_t - y \tag{1}$$

leads to the system of first-order equations

$$u'_t = atf(t)v, \qquad v'_t = btf(t)u.$$

The general solution of this system has the form

$$\text{with } ab > 0: \quad \begin{cases} u(t) = C_1 a \exp\left(\sqrt{ab}\int tf(t)\,dt\right) + C_2 a \exp\left(-\sqrt{ab}\int tf(t)\,dt\right), \\ v(t) = C_1\sqrt{ab}\,\exp\left(\sqrt{ab}\int tf(t)\,dt\right) - C_2\sqrt{ab}\,\exp\left(-\sqrt{ab}\int tf(t)\,dt\right), \end{cases}$$

$$\text{with } ab < 0: \quad \begin{cases} u(t) = C_1 a \cos\left(\sqrt{|ab|}\int tf(t)\,dt\right) + C_2 a \sin\left(\sqrt{|ab|}\int tf(t)\,dt\right), \\ v(t) = -C_1\sqrt{|ab|}\,\sin\left(\sqrt{|ab|}\int tf(t)\,dt\right) + C_2\sqrt{|ab|}\,\cos\left(\sqrt{|ab|}\int tf(t)\,dt\right), \end{cases} \tag{2}$$

where C_1 and C_2 are arbitrary constants. On substituting (2) into (1) and on integrating the resulting relation, one arrives at the solution of the original system:

$$x = C_3 t + t\int \frac{u(t)}{t^2}\,dt, \qquad y = C_4 t + t\int \frac{v(t)}{t^2}\,dt,$$

where C_3 and C_4 are arbitrary constants.