16. $t^2x''_{tt} + a_1tx'_t + b_1ty'_t + c_1x + d_1y = 0, \quad t^2y''_{tt} + a_2tx'_t + b_2ty'_t + c_2x + d_2y = 0.$

Linear system homogeneous in the independent variable (Euler type).

1°. The general solution is determined by a linear combination of linearly independent particular solutions sought in the form of power-law functions,

$$x = A|t|^k, \quad y = B|t|^k,$$

by the method of undetermined coefficients. On substituting these expressions into the original system and collecting coefficients of A and B, one obtains a system for A and B:

$$[k^2 + (a_1 - 1)k + c_1]A + (b_1k + d_1)B = 0,$$
$$(a_2k + c_2)A + [k^2 + (b_2 - 1)k + d_2]B = 0.$$

The determinant of this system must vanish for nontrivial solutions to exist. Hence follows the characteristic equation for the exponent k:

$$[k^2 + (a_1 - 1)k + c_1][k^2 + (b_2 - 1)k + d_2] - (b_1k + d_1)(a_2k + c_2) = 0.$$

If the roots of this equation, $k_1, \dots, k_4$, are all distinct, then the general solution of the original system of differential equations is expressed as

$$x = -C_1(b_1k_1 + d_1)|t|^{k_1} - C_2(b_1k_2 + d_1)|t|^{k_2} - C_3(b_1k_1 + d_1)|t|^{k_3} - C_4(b_1k_4 + d_1)|t|^{k_4},$$
$$y = C_1[k_1^2 + (a_1 - 1)k_1 + c_1]|t|^{k_1} + C_2[k_2^2 + (a_1 - 1)k_2 + c_1]|t|^{k_2}$$
$$+ C_3[k_3^2 + (a_1 - 1)k_3 + c_1]|t|^{k_3} + C_4[k_4^2 + (a_1 - 1)k_4 + c_1]|t|^{k_4},$$

where $C_1, \dots, C_4$ are arbitrary constants.

2°. The substitution $t = \sigma e^\tau$ $(\sigma \neq 0)$ leads to the system of constant-coefficient linear differential equations

$$x''_{\tau\tau} + (a_1 - 1)x'_\tau + b_1ty'_\tau + c_1x + d_1y = 0,$$
$$y''_{\tau\tau} + a_2x'_\tau + (b_2 - 1)y'_\tau + c_2x + d_2y = 0.$$

