18. $x''_{tt} = f(t)(tx'_t - x) + g(t)(ty'_t - y), \quad y''_{tt} = h(t)(tx'_t - x) + p(t)(ty'_t - y).$

The transformation

$$u = tx_t - x, \qquad v = ty'_t - y \tag{1}$$

leads to the linear system of first-order equations

$$u'_t = tf(t)u + tg(t)v, \qquad v'_t = th(t)u + tp(t)v. \tag{2}$$

In order to obtain its general solution, it suffices to know a particular solution of this system (see equation 1.7 in the current section).

For solutions to some systems of the form 92), see equations 1.3–1.6.

If all the functions in (2) are proportional,

$$f(t) = a\varphi(t), \quad g(t) = b\varphi(t), \quad h(t) = c\varphi(t), \quad p(t) = d\varphi(t),$$

then the introduction of the new independent variable $\tau = \int t\varphi(t)\,dt$ leads to a constant-coefficient system of the form 1.1.

2°. Suppose a solution of system (2),

$$u = u(t, C_1, C_2), \qquad v = v(t, C_1, C_2), \tag{3}$$

where C_1 and C_2 are arbitrary constants, is known. Then, on substituting (3) into (1) and on integrating the resulting equation, one obtains a solution of the original system in the form

$$x = C_3 t + t\int \frac{u(t, C_1, C_2)}{t^2}\,dt, \qquad y = C_4 t + t\int \frac{v(t, C_1, C_2)}{t^2}\,dt.$$

where C_3 and C_4 are arbitrary constants.

