



Systems of Ordinary Differential Equations > Linear Systems of Three and More Equations

$$2. \quad x'_t = cy - bz, \quad y'_t = az - cx, \quad z'_t = bx - ay.$$

1°. First integrals:

$$ax + by + cz = A, \tag{1}$$

$$x^2 + y^2 + z^2 = B^2, \tag{2}$$

where A and B are arbitrary constants. It follows from these integrals that the integral lines are circles formed by the intersection of the planes (1) and spheres (2).

2°. Solution:

$$x = aC_0 + kC_1 \cos(kt) + (cC_2 - bC_3) \sin(kt),$$

$$y = bC_0 + kC_2 \cos(kt) + (aC_3 - cC_1) \sin(kt),$$

$$z = cC_0 + kC_3 \cos(kt) + (bC_1 - aC_2) \sin(kt),$$

where $k = \sqrt{a^2 + b^2 + c^2}$ and the four constants of integration, C_1, \dots, C_4 , are constrained by a single relation,

$$aC_1 + bC_2 + cC_3 = 0.$$

Reference

Kamke, E., *Differentialgleichungen: Lösungsmethoden und Lösungen, I. Gewöhnliche Differentialgleichungen*, B. G. Teubner, Leipzig, 1977.