



Systems of Ordinary Differential Equations > Linear Systems of Three and More Equations

$$6. \quad x'_k = a_{k1}x_1 + a_{k2}x_2 + \cdots + a_{kn}x_n; \quad k = 1, 2, \dots, n.$$

*System of  $n$  first-order constant-coefficient linear nonhomogeneous differential equations.*

The general solution of the homogeneous system of differential equations is a linear combination of linearly independent particular solutions sought by the method of undetermined coefficients in the form of exponentials,

$$x_k = A_k e^{\lambda t}; \quad k = 1, 2, \dots, n.$$

On substituting these expressions into the system and collecting the coefficients of the unknowns  $A_k$ , one arrives at the linear homogeneous algebraic system of equations

$$a_{k1}A_1 + a_{k2}A_2 + \cdots + (a_{kk} - \lambda)A_k + \cdots + a_{kn}A_n = 0; \quad k = 1, 2, \dots, n.$$

The determinant of the system must vanish for a nontrivial solution  $A_1, \dots, A_n$  to exist. This requirement yields the characteristic equation for determining the exponent  $\lambda$ .

### References

- Kamke, E.**, *Differentialgleichungen: Lösungsmethoden und Lösungen, I, Gewöhnliche Differentialgleichungen*, B. G. Teubner, Leipzig, 1977.
- Boyce, W. E. and DiPrima, R. C.**, *Elementary Differential Equations and Boundary Value Problems, 8th Edition*, Wiley, New York, 2004.