Systems of Ordinary Differential Equations > Nonlinear Systems of Two Equations

6. $\quad x_{tt}'' = x f(ax - by) + g(ax - by), \quad y_{tt}'' = y f(ax - by) + h(ax - by).$

Let us multiply the first equation by a and the second one by $-b$ and let us add the results together to obtain the autonomous equation

$$z_{tt}'' = z f(z) + a g(z) - b h(z), \qquad z = ax - by. \tag{1}$$

This equation will be treated in conjunction with the first equation of the original system,

$$x_{tt}'' = x f(z) + g(z). \tag{2}$$

Equation (1) can be treated separately and its solution can be written out in implicit form (see A. D. Polyanin & V. F. Zaitsev, 1995). The function $x = x(t)$ is found by solving the linear equation (2), and the function $y = y(t)$ is then determined by the formula $y = (ax - z)/b$.