8. $x''_{tt} = kxr^{-3}$, $y''_{tt} = kyr^{-3}$, **where** $r = \sqrt{x^2 + y^2}$.

Equation of motion of point mass in the xy-plane under gravitational force.

On proceeding to polar coordinates by the formulas

$$x = r\cos\varphi, \quad y = r\sin\varphi, \quad r = r(t), \quad \varphi = \varphi(t),$$

one can obtain the first integrals

$$r^2\varphi'_t = C_1, \quad (r'_t)^2 + r^2(\varphi'_t)^2 = -2kr^{-1} + C_2,$$

where C_1 and C_2 are arbitrary constants. Integrating further, with $C_1 \neq 0$, yields

$$r[C\cos(\varphi - \varphi_0) - k] = C_1^2, \qquad C^2 = C_1^2C_2 + k^2.$$

This equation describes conical sections. The function $\varphi(t)$ can be determined using either first integral.

Reference

Kamke, E., *Differentialgleichungen: Lösungsmethoden und Lösungen, I, Gewöhnliche Differentialgleichungen*, B. G. Teubner, Leipzig, 1977.

