9. $x''_{tt} = xf(r), \quad y''_{tt} = yf(r), \quad$ **where** $r = \sqrt{x^2 + y^2}$.

Equation of motion of a point mass in the xy-plane under central force.

On proceeding to polar coordinates by the formulas

$$x = r\cos\varphi, \quad y = r\sin\varphi, \quad r = r(t), \quad \varphi = \varphi(t),$$

one can obtain the first integrals

$$r^2\varphi'_t = C_1, \quad (r'_t)^2 + r^2(\varphi'_t)^2 = 2\int rf(r)\,dr + C_2,$$

where C_1 and C_2 are arbitrary constants. Integrating further yields

$$t + C_3 = \pm\int\frac{r\,dr}{\sqrt{2r^2F(r) + r^2C_2 - C_1^2}}, \quad \varphi = C_1\int\frac{dt}{r} + C_4, \qquad (*)$$

where C_3 and C_4 are arbitrary constants, and

$$F(r) = \int rf(r)\,dr$$

In the last relation of $(*)$, it is assumed that the dependence $r = r(t)$ is obtained by solving the first equation of $(*)$ for r.

Reference

Kamke, E., *Differentialgleichungen: Lösungsmethoden und Lösungen, I, Gewöhnliche Differentialgleichungen*, B. G. Teubner, Leipzig, 1977.

