10. $x''_{tt} = xf(x^2 + y^2, y/x) - yg(y/x), \quad y''_{tt} = yf(x^2 + y^2, y/x) + xg(y/x).$

Particular solution:

$$u = r(t)\cos\beta, \qquad w = r(t)\sin\beta,$$

where the constant β is determined by the transcendental equation

$$g(\tan\beta) = 0$$

and the function $r = r(t)$ satisfies the solvable autonomous second-order equation

$$r''_{tt} = rf(r^2, \tan\beta).$$

For $g(z) \equiv 0$, β is an arbitrary constant.

