



Systems of Ordinary Differential Equations > Nonlinear Systems of Two Equations

13. $x''_{tt} = \frac{1}{x^3} F\left(\frac{x}{\varphi(t)}, \frac{y}{\varphi(t)}\right), \quad y''_{tt} = \frac{1}{y^3} G\left(\frac{x}{\varphi(t)}, \frac{y}{\varphi(t)}\right), \quad \varphi(t) = \sqrt{at^2 + bt + c}.$

1°. The transformation

$$\tau = \int \frac{dt}{\varphi^2(t)}, \quad u = \frac{x}{\varphi(t)}, \quad v = \frac{y}{\varphi(t)}$$

leads to the autonomous system of equations

$$\begin{aligned} u''_{\tau\tau} + \left(ac - \frac{1}{4}b^2\right)u &= u^{-3}F(u, v), \\ v''_{\tau\tau} + \left(ac - \frac{1}{4}b^2\right)v &= v^{-3}G(u, v). \end{aligned}$$

2°. Particular solutions:

$$x = A\sqrt{at^2 + bt + c}, \quad y = B\sqrt{at^2 + bt + c},$$

where A and B are the roots of the system of algebraic (transcendental) equations

$$\begin{aligned} \left(ac - \frac{1}{4}b^2\right)A^4 &= F(A, B), \\ \left(ac - \frac{1}{4}b^2\right)B^4 &= G(A, B). \end{aligned}$$