14. $x''_{tt} = f(y'_t/x'_t), \quad y''_{tt} = g(y'_t/x'_t).$

1°. The transformation

$$u = x'_t, \qquad w = y'_t \tag{1}$$

leads to the system of first-order equations

$$u'_t = f(w/u), \qquad w'_t = g(w/u). \tag{2}$$

On eliminating t, one arrives at a homogeneous first-order equation, whose solution is expressed as

$$\int \frac{f(\xi)\,d\xi}{g(\xi) - \xi f(\xi)} = \ln|u| + C, \qquad \xi = \frac{w}{u}, \tag{3}$$

where C is an arbitrary constant. Solving (3) for w yields $w = w(u, C)$. On substituting this expression into the first equation in (2), one can obtain $u = u(t)$ and then $w = w(t)$. Eventually, one can obtain $x = x(t)$ and $y = y(t)$ by simple integration.

2°. *The Suslov problem.* The problem of a point particle sliding down an inclined rough plane is described by the equations

$$x''_{tt} = 1 - \frac{kx'_t}{\sqrt{(x'_t)^2 + (y'_t)^2}}, \qquad y''_{tt} = -\frac{ky'_t}{\sqrt{(x'_t)^2 + (y'_t)^2}},$$

which is a special case of the original system with

$$f(z) = 1 - \frac{k}{\sqrt{1+z^2}}, \qquad g(z) = -\frac{kz}{\sqrt{1+z^2}}.$$

The solution of the Cauchy problem with the initial conditions

$$x(0) = y(0) = x'_t(0) = 0, \qquad y'_t(0) = 1$$

and with $k = 1$ results in the following dependences $x(t)$ and $y(t)$ in parametric form:

$$x = -\tfrac{1}{16} + \tfrac{1}{16}\xi^4 - \tfrac{1}{4}\ln\xi, \quad y = \tfrac{2}{3} - \tfrac{1}{2}\xi - \tfrac{1}{6}\xi^3, \quad t = \tfrac{1}{4} - \tfrac{1}{4}\xi^2 - \tfrac{1}{2}\ln\xi \qquad (0 \le \xi \le 1).$$

Reference

Klimov, D. M. and Zhuravlev, V. Ph., *Group-Theoretic Methods in Mechanics and Applied Mathematics*, Taylor & Francis, London, 2002.

