17. $\boldsymbol{x''_{tt} = F(t, tx'_t - x, ty'_t - y), \quad y''_{tt} = G(t, tx'_t - x, ty'_t - y).}$

1°. The transformation

$$u = tx_t - x, \qquad v = ty'_t - y \tag{1}$$

leads to the system of first-order equations

$$u'_t = tF(t, u, v), \qquad v'_t = tG(t, u, v). \tag{2}$$

2°. Suppose a solution of system (2),

$$u = u(t, C_1, C_2), \qquad v = v(t, C_1, C_2), \tag{3}$$

where C_1 and C_2 are arbitrary constants, is known. Then, on substituting (3) into (1) and integrating, one obtains a solution of the original system in the form

$$x = C_3 t + t \int \frac{u(t, C_1, C_2)}{t^2}\,dt, \qquad y = C_4 t + t \int \frac{v(t, C_1, C_2)}{t^2}\,dt.$$

3°. If the functions F and G are independent of t, then, having eliminated t from system (2), one arrives at the first-order equation

$$g(u, v) u'_v = F(u, v).$$

⊙ *Reference*: A. D. Polyanin, *EqWorld*, 2004 (Private communication, received 23 April 2004).

