



Systems of Ordinary Differential Equations > Nonlinear Systems of Three and More Equations

5. $x'_t = czF_2 - byF_3$, $y'_t = axF_3 - czF_1$, $z'_t = byF_1 - axF_2$, where $F_n = F_n(x, y, z, t)$.

1°. First integral:

$$ax^2 + by^2 + cz^2 = C_1,$$

where C is an arbitrary constant.

2°. Suppose the function F_n is independent of t : $F_n = F_n(x, y, z)$. Then, on eliminating t and z from the first two equations of the system (with the above integral), one arrives at the first-order equation

$$\frac{dy}{dx} = \frac{axF_3(x, y, z) - czF_1(x, y, z)}{czF_2(x, y, z) - byF_3(x, y, z)}, \quad \text{where } z = \pm \sqrt{\frac{1}{c}(C_1 - ax^2 - by^2)}.$$