



Systems of Ordinary Differential Equations > Nonlinear Systems of Three and More Equations

10. $x''_{tt} = F_1, \quad y''_{tt} = F_2, \quad z''_{tt} = F_3, \quad \text{where} \quad F_n = F_n(t, tx'_t - x, ty'_t - y, tz'_t - z).$

1°. The transformation

$$u = tx_t - x, \quad v = ty'_t - y, \quad w = tz'_t - z \quad (1)$$

leads to the system of first-order equations

$$u'_t = tF_1(t, u, v, w), \quad v'_t = tF_2(t, u, v, w), \quad w'_t = tF_3(t, u, v, w). \quad (2)$$

2°. Suppose a solution to system (2),

$$u(t) = u(t, C_1, C_2, C_3), \quad v(t) = v(t, C_1, C_2, C_3), \quad w(t) = w(t, C_1, C_2, C_3), \quad (3)$$

where $C_1, C_2,$ and C_3 are arbitrary constants, is known. Then, on substituting (3) into (1) and on integrating the resulting relation, one arrives at a solution of the original system in the form

$$x = C_4t + t \int \frac{u(t)}{t^2} dt, \quad y = C_5t + t \int \frac{v(t)}{t^2} dt, \quad z = C_6t + t \int \frac{w(t)}{t^2} dt,$$

where $C_4, C_5,$ and C_6 are arbitrary constants.

⊙ *Reference:* A. D. Polyanin, *EqWorld*, 2004 (Private communication, received 23 April 2004).