1. $\dfrac{\partial u}{\partial t} = a\dfrac{\partial^2 u}{\partial x^2} + b_1 u + c_1 w, \qquad \dfrac{\partial w}{\partial t} = a\dfrac{\partial^2 w}{\partial x^2} + b_2 u + c_2 w.$

Second-order constant-coefficient linear parabolic system.

Solution:

$$u = \frac{b_1 - \lambda_2}{b_2(\lambda_1 - \lambda_2)} e^{\lambda_1 t}\theta_1 - \frac{b_1 - \lambda_1}{b_2(\lambda_1 - \lambda_2)} e^{\lambda_2 t}\theta_2,$$
$$w = \frac{1}{\lambda_1 - \lambda_2}\bigl(e^{\lambda_1 t}\theta_1 - e^{\lambda_2 t}\theta_2\bigr),$$

where λ_1 and λ_2 are roots of the quadratic equation

$$\lambda^2 - (b_1 + c_2)\lambda + b_1 c_2 - b_2 c_1 = 0,$$

and the functions $\theta_n = \theta_n(x,t)$ satisfy the linear heat equations

$$\frac{\partial \theta_1}{\partial t} = a\frac{\partial^2 \theta_1}{\partial x^2}, \qquad \frac{\partial \theta_2}{\partial t} = a\frac{\partial^2 \theta_2}{\partial x^2}.$$

