2. $\dfrac{\partial u}{\partial t} = a\dfrac{\partial^2 u}{\partial x^2} + f_1(t)u + g_1(t)w, \qquad \dfrac{\partial w}{\partial t} = a\dfrac{\partial^2 w}{\partial x^2} + f_2(t)u + g_2(t)w.$

Second-order variable-coefficient linear parabolic system.

Solution:

$$u = \varphi_1(t)U(x,t) + \varphi_2(t)W(x,t),$$
$$w = \psi_1(t)U(x,t) + \psi_2(t)W(x,t),$$

where the pairs of functions $\varphi_1 = \varphi_1(t)$, $\psi_1 = \psi_1(t)$ and $\varphi_2 = \varphi_2(t)$, $\psi_2 = \psi_2(t)$ are linearly independent (fundamental) solutions of the system of linear ordinary differential equations

$$\varphi'_t = f_1(t)\varphi + g_1(t)\psi,$$
$$\psi'_t = f_2(t)\varphi + g_2(t)\psi,$$

and the functions $U = U(x,t)$ and $W = W(x,t)$ satisfy the linear heat equations

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2}, \qquad \frac{\partial W}{\partial t} = a\frac{\partial^2 W}{\partial x^2}.$$

