



Exact Solutions > Systems of Partial Differential Equations > Linear Systems of Two Second-Order Equations

$$3. \quad \frac{\partial^2 u}{\partial t^2} = k \frac{\partial^2 u}{\partial x^2} + a_1 u + b_1 w, \quad \frac{\partial^2 w}{\partial t^2} = k \frac{\partial^2 w}{\partial x^2} + a_2 u + b_2 w.$$

Second-order constant-coefficient linear hyperbolic system.

Solution:

$$u = \frac{a_1 - \lambda_2}{a_2(\lambda_1 - \lambda_2)} \theta_1 - \frac{a_1 - \lambda_1}{a_2(\lambda_1 - \lambda_2)} \theta_2, \quad w = \frac{1}{\lambda_1 - \lambda_2} (\theta_1 - \theta_2),$$

where λ_1 and λ_2 are roots of the quadratic equation

$$\lambda^2 - (a_1 + b_2)\lambda + a_1 b_2 - a_2 b_1 = 0,$$

and the functions $\theta_n = \theta_n(x, t)$ satisfy the linear Klein–Gordon equations

$$\frac{\partial^2 \theta_1}{\partial t^2} = k \frac{\partial^2 \theta_1}{\partial x^2} + \lambda_1 \theta_1, \quad \frac{\partial^2 \theta_2}{\partial t^2} = k \frac{\partial^2 \theta_2}{\partial x^2} + \lambda_2 \theta_2.$$

Second-order constant-coefficient linear hyperbolic system