4. $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = a_1 u + b_1 w, \qquad \dfrac{\partial^2 w}{\partial x^2} + \dfrac{\partial^2 w}{\partial y^2} = a_2 u + b_2 w.$

Second-order constant-coefficient linear elliptic system.

Solution:

$$u = \frac{a_1 - \lambda_2}{a_2(\lambda_1 - \lambda_2)}\theta_1 - \frac{a_1 - \lambda_1}{a_2(\lambda_1 - \lambda_2)}\theta_2, \qquad w = \frac{1}{\lambda_1 - \lambda_2}\bigl(\theta_1 - \theta_2\bigr),$$

where λ_1 and λ_2 are roots of the quadratic equation

$$\lambda^2 - (a_1 + b_2)\lambda + a_1 b_2 - a_2 b_1 = 0,$$

and the functions $\theta_n = \theta_n(x, y)$ satisfies the linear Helmholtz equation

$$\frac{\partial^2 \theta_1}{\partial x^2} + \frac{\partial^2 \theta_1}{\partial y^2} = \lambda_1 \theta_1, \qquad \frac{\partial^2 \theta_2}{\partial x^2} + \frac{\partial^2 \theta_2}{\partial y^2} = \lambda_2 \theta_2.$$

