1. $\dfrac{\partial u}{\partial t} = a\dfrac{\partial^2 u}{\partial x^2} + u\exp\Big(k\dfrac{w}{u}\Big)f(u), \qquad \dfrac{\partial w}{\partial t} = a\dfrac{\partial^2 w}{\partial x^2} + \exp\Big(k\dfrac{w}{u}\Big)[wf(u)+g(u)].$

Solution:

$$u = y(\xi), \quad w = -\frac{2}{k}\ln|bx|\,y(\xi) + z(\xi), \quad \xi = \frac{x + C_3}{\sqrt{C_1 t + C_2}},$$

where C_1, C_2, C_3, and b are arbitrary constants, and the functions $y = y(\xi)$ and $z = z(\xi)$ are determined by the system of ordinary differential equations

$$ay''_{\xi\xi} + \frac{1}{2}C_1\xi y'_\xi + \frac{1}{b^2\xi^2}y\exp\Big(k\frac{z}{y}\Big)f(y) = 0,$$

$$az''_{\xi\xi} + \frac{1}{2}C_1\xi z'_\xi - \frac{4a}{k\xi}y'_\xi + \frac{2a}{k\xi^2}y + \frac{1}{b^2\xi^2}\exp\Big(k\frac{z}{y}\Big)[zf(y) + g(y)] = 0.$$

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