3. $\dfrac{\partial u}{\partial t} = a\dfrac{\partial^2 u}{\partial x^2} + e^{\lambda u} f(\lambda u - \sigma w), \qquad \dfrac{\partial w}{\partial t} = b\dfrac{\partial^2 w}{\partial x^2} + e^{\sigma w} g(\lambda u - \sigma w).$

1°. Solution:

$$u = y(\xi) - \frac{1}{\lambda}\ln(C_1 t + C_2), \qquad w = z(\xi) - \frac{1}{\sigma}\ln(C_1 t + C_2), \qquad \xi = \frac{x + C_3}{\sqrt{C_1 t + C_2}},$$

where C_1, C_2, and C_3 are arbitrary constants, and the functions $y = y(\xi)$ and $z = z(\xi)$ are determined by the system of ordinary differential equations

$$ay''_{\xi\xi} + \frac{1}{2}C_1\xi y'_\xi + \frac{C_1}{\lambda} + e^{\lambda y} f(\lambda y - \sigma z) = 0,$$
$$bz''_{\xi\xi} + \frac{1}{2}C_1\xi z'_\xi + \frac{C_1}{\sigma} + e^{\sigma z} g(\lambda y - \sigma z) = 0.$$

2°. Solution with $b = a$:

$$u = \theta(x,t), \qquad w = \frac{\lambda}{\sigma}\theta(x,t) - \frac{k}{\sigma},$$

where k is a root of the algebraic (transcendental) equation

$$\lambda f(k) = \sigma e^{-k} g(k),$$

and the function $\theta = \theta(x,t)$ is determined by the differential equation

$$\frac{\partial \theta}{\partial t} = a\frac{\partial^2 \theta}{\partial x^2} + f(k)e^{\lambda\theta}.$$

See the “Handbook of Nonlinear Partial Differential Equations” by A. D. Polyanin & V. F. Zaitsev (2004), for exact solutions of this equation.

Reference

Polyanin, A. D., Exact solutions of nonlinear systems of reaction-diffusion equations and mathematical biology equations, *Theor. Found. Chem. Eng.,* Vol. 37, No. 6, 2004.

