5. $\dfrac{\partial u}{\partial t} = a\dfrac{\partial^2 u}{\partial x^2} + uf\left(\dfrac{u}{w}\right), \quad \dfrac{\partial w}{\partial t} = b\dfrac{\partial^2 w}{\partial x^2} + wg\left(\dfrac{u}{w}\right).$

1°. Multiplicative separable solution:

$$u = [C_1 \sin(kx) + C_2 \cos(kx)]\varphi(t),$$
$$w = [C_1 \sin(kx) + C_2 \cos(kx)]\psi(t),$$

where C_1, C_2, and k are arbitrary constants, and the functions $\varphi = \varphi(t)$ and $\psi = \psi(t)$ are determined by the system of ordinary differential equations

$$\varphi'_t = -ak^2\varphi + \varphi f(\varphi/\psi),$$
$$\psi'_t = -bk^2\psi + \psi g(\varphi/\psi).$$

2°. Multiplicative separable solution:

$$u = [C_1 \exp(kx) + C_2 \exp(-kx)]U(t),$$
$$w = [C_1 \exp(kx) + C_2 \exp(-kx)]W(t),$$

where C_1, C_2, and k are arbitrary constants, and the functions $U = U(t)$ and $W = W(t)$ are determined by the system of ordinary differential equations

$$U'_t = ak^2U + Uf(U/W),$$
$$W'_t = bk^2W + Wg(U/W).$$

3°. Degenerate solution:

$$u = (C_1x + C_2)U(t),$$
$$w = (C_1x + C_2)W(t),$$

where C_1 and C_2, and the functions $U = U(t)$ and $W = W(t)$ are determined by the system of ordinary differential equations

$$U'_t = Uf(U/W),$$
$$W'_t = Wg(U/W).$$

This autonomous system can be integrated, since it is reduced, on eliminating t, to a homogeneous first-order equation (the corresponding systems of Items 1° and 2° are integrated likewise).

4°. Multiplicative separable solution:

$$u = e^{-\lambda t}y(x), \qquad w = e^{-\lambda t}z(x),$$

where λ is an arbitrary constant and the functions $y = y(x)$ and $z = z(x)$ are determined by the system of ordinary differential equations

$$ay''_{xx} + \lambda y + yf(y/z) = 0,$$
$$bz''_{xx} + \lambda z + zg(y/z) = 0.$$

5°. Solution(generalizes the solution of Item 4°):

$$u = e^{kx-\lambda t}y(\xi), \qquad w = e^{kx-\lambda t}z(\xi), \qquad \xi = \beta x - \gamma t,$$

where k, λ, β, and γ are arbitrary constants, and the functions $y = y(\xi)$ and $z = z(\xi)$ are determined by the system of ordinary differential equations

$$a\beta^2 y''_{\xi\xi} + (2ak\beta + \gamma)y'_\xi + (ak^2 + \lambda)y + yf(y/z) = 0,$$
$$b\beta^2 z''_{\xi\xi} + (2bk\beta + \gamma)z'_\xi + (bk^2 + \lambda)z + zg(y/z) = 0.$$

To the special case $k = \lambda = 0$ there corresponds a traveling-wave solution. For $k = \gamma = 0$ and $\beta = 1$, we have the solution of Item 3°.

Reference

Polyanin, A. D., Exact solutions of nonlinear systems of reaction-diffusion equations and mathematical biology equations, *Theor. Found. Chem. Eng.,* Vol. 37, No. 6, 2004.

