6. $\dfrac{\partial u}{\partial t} = a\dfrac{\partial^2 u}{\partial x^2} + uf\left(\dfrac{u}{w}\right) + g\left(\dfrac{u}{w}\right), \quad \dfrac{\partial w}{\partial t} = a\dfrac{\partial^2 w}{\partial x^2} + wf\left(\dfrac{u}{w}\right) + h\left(\dfrac{u}{w}\right).$

Let k be a root of the algebraic (transcendental) equation

$$g(k) = kh(k).$$

1°. Solution if $f(k) \neq 0$:

$$u(x,t) = k\left(\exp[f(k)t]\theta(x,t) - \frac{h(k)}{f(k)}\right), \quad w(x,t) = \exp[f(k)t]\theta(x,t) - \frac{h(k)}{f(k)},$$

where the function $\theta = \theta(x,t)$ satisfies the linear heat equation

$$\frac{\partial \theta}{\partial t} = a\frac{\partial^2 \theta}{\partial x^2}. \qquad (1)$$

2°. Solution if $f(k) = 0$:

$$u(x,t) = k[\theta(x,t) + h(k)t], \quad w(x,t) = \theta(x,t) + h(k)t,$$

where the function $\theta = \theta(x,t)$ satisfies the linear heat equation (1).

Reference

Polyanin, A. D., Exact solutions of nonlinear systems of reaction-diffusion equations and mathematical biology equations, *Theor. Found. Chem. Eng.*, Vol. 37, No. 6, 2004.

