7. $\dfrac{\partial u}{\partial t} = a\dfrac{\partial^2 u}{\partial x^2} + uf\left(\dfrac{u}{w}\right) + \dfrac{u}{w}h\left(\dfrac{u}{w}\right), \quad \dfrac{\partial w}{\partial t} = a\dfrac{\partial^2 w}{\partial x^2} + wg\left(\dfrac{u}{w}\right) + h\left(\dfrac{u}{w}\right).$

Solution:

$$u = \varphi(t)G(t)\left[\theta(x,t) + \int \frac{h(\varphi)}{G(t)}\,dt\right], \quad w = G(t)\left[\theta(x,t) + \int \frac{h(\varphi)}{G(t)}\,dt\right], \quad G(t) = \exp\left[\int g(\varphi)\,dt\right],$$

where the function $\varphi = \varphi(t)$ is determined by the separable nonlinear ordinary differential equation

$$\varphi'_t = [f(\varphi) - g(\varphi)]\varphi, \tag{1}$$

and the function $\theta = \theta(x,t)$ satisfies the linear heat equation

$$\frac{\partial \theta}{\partial t} = a\frac{\partial^2 \theta}{\partial x^2}.$$

The general solution of equation (1) is written out in implicit form

$$\int \frac{d\varphi}{[f(\varphi) - g(\varphi)]\varphi} = t + C.$$

Reference

Polyanin, A. D., Exact solutions of nonlinear systems of reaction-diffusion equations and mathematical biology equations, *Theor. Found. Chem. Eng.*, Vol. 37, No. 6, 2004.

