8. $\dfrac{\partial u}{\partial t} = a\dfrac{\partial^2 u}{\partial x^2} + u^3 f\left(\dfrac{u}{w}\right), \qquad \dfrac{\partial w}{\partial t} = a\dfrac{\partial^2 w}{\partial x^2} + u^3 g\left(\dfrac{u}{w}\right).$

Solution:

$$u = (x + A)\varphi(z), \qquad w = (x + A)\psi(z), \qquad z = t + \frac{1}{6a}(x + A)^2 + B,$$

where A and B are arbitrary constants, and the functions $\varphi = \varphi(z)$ and $\psi = \psi(z)$ are determined by the system of ordinary differential equations

$$\varphi''_{zz} + 9a\varphi^3 f(\varphi/\psi) = 0,$$
$$\psi''_{zz} + 9a\varphi^3 g(\varphi/\psi) = 0.$$

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