9. $\dfrac{\partial u}{\partial t}=\dfrac{\partial^2 u}{\partial x^2}+au-u^3 f\left(\dfrac{u}{w}\right),\qquad \dfrac{\partial w}{\partial t}=\dfrac{\partial^2 w}{\partial x^2}+aw-u^3 g\left(\dfrac{u}{w}\right).$

1°. Solution with $a>0$:

$$u=\left[C_1\exp\left(\tfrac{1}{2}\sqrt{2a}\,x+\tfrac{3}{2}at\right)-C_2\exp\left(-\tfrac{1}{2}\sqrt{2a}\,x+\tfrac{3}{2}at\right)\right]\varphi(z),$$
$$w=\left[C_1\exp\left(\tfrac{1}{2}\sqrt{2a}\,x+\tfrac{3}{2}at\right)-C_2\exp\left(-\tfrac{1}{2}\sqrt{2a}\,x+\tfrac{3}{2}at\right)\right]\psi(z),$$
$$z=C_1\exp\left(\tfrac{1}{2}\sqrt{2a}\,x+\tfrac{3}{2}at\right)+C_2\exp\left(-\tfrac{1}{2}\sqrt{2a}\,x+\tfrac{3}{2}at\right)+C_3,$$

where C_1, C_2, and C_3 are arbitrary constants, and the functions $\varphi=\varphi(z)$ and $\psi=\psi(z)$ are determined by the system of ordinary differential equations

$$a\varphi''_{zz}=2\varphi^3 f(\varphi/\psi),$$
$$a\psi''_{zz}=2\varphi^3 g(\varphi/\psi).$$

2°. Solution with $a<0$:

$$u=\exp\left(\tfrac{3}{2}at\right)\sin\left(\tfrac{1}{2}\sqrt{2|a|}\,x+C_1\right)U(\xi),$$
$$w=\exp\left(\tfrac{3}{2}at\right)\sin\left(\tfrac{1}{2}\sqrt{2|a|}\,x+C_1\right)W(\xi),$$
$$\xi=\exp\left(\tfrac{3}{2}at\right)\cos\left(\tfrac{1}{2}\sqrt{2|a|}\,x+C_1\right)+C_2,$$

where C_1 and C_2 are arbitrary constants, and the functions $U=U(\xi)$ and $W=W(\xi)$ are determined by the system of ordinary differential equations

$$aU''_{\xi\xi}=-2U^3 f(U/W),$$
$$aW''_{\xi\xi}=-2U^3 g(U/W).$$

Reference

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