10. $\dfrac{\partial u}{\partial t}=a\dfrac{\partial^2 u}{\partial x^2}+u^n f\left(\dfrac{u}{w}\right),\quad \dfrac{\partial w}{\partial t}=b\dfrac{\partial^2 w}{\partial x^2}+w^n g\left(\dfrac{u}{w}\right).$

For $f(z)=kz^{-m}$ and $g(z)=-kz^{n-m}$, the system describes a chemical reaction of order n (order $n-m$ in the component u and order m in the component w).

1°. Self-similar solution:

$$u=(C_1t+C_2)^{\frac{1}{1-n}}y(\xi),\qquad w=(C_1t+C_2)^{\frac{1}{1-n}}z(\xi),\qquad \xi=\frac{x+C_3}{\sqrt{C_1t+C_2}},$$

where C_1, C_2, and C_3 are arbitrary constants, and the functions $y=y(\xi)$ and $z=z(\xi)$ are determined by the system of ordinary differential equations

$$ay''_{\xi\xi}+\frac{1}{2}C_1\xi y'_\xi+\frac{C_1}{n-1}y+y^n f\left(\frac{y}{z}\right)=0,$$
$$bz''_{\xi\xi}+\frac{1}{2}C_1\xi z'_\xi+\frac{C_1}{n-1}z+z^n g\left(\frac{y}{z}\right)=0.$$

2°. Solution with $b=a$:

$$u(x,t)=k\theta(x,t),\qquad w(x,t)=\theta(x,t),$$

where k is a root of the algebraic (transcendental) equation

$$k^{n-1}f(k)=g(k),$$

and the function $\theta=\theta(x,t)$ satisfies the heat equation with power-law nonlinearity

$$\frac{\partial\theta}{\partial t}=a\frac{\partial^2\theta}{\partial x^2}+g(k)\theta^n.$$

Reference

Polyanin, A. D., Exact solutions of nonlinear systems of reaction-diffusion equations and mathematical biology equations, *Theor. Found. Chem. Eng.*, Vol. 37, No. 6, 2004.

