12. $$\frac{\partial u}{\partial t} = a\frac{\partial^2 u}{\partial x^2} + uf\left(\frac{w}{u}\right) - wg\left(\frac{w}{u}\right) + \frac{u}{\sqrt{u^2+w^2}}h\left(\frac{w}{u}\right),$$
$$\frac{\partial w}{\partial t} = a\frac{\partial^2 w}{\partial x^2} + wf\left(\frac{w}{u}\right) + ug\left(\frac{w}{u}\right) + \frac{w}{\sqrt{u^2+w^2}}h\left(\frac{w}{u}\right).$$

Solution:

$$u = r(x,t)\cos\varphi(t), \qquad w = r(x,t)\sin\varphi(t),$$

where the function $\varphi = \varphi(t)$ is determined by the separable first-order ordinary differential equation

$$\varphi'_t = g(\tan\varphi),$$

and the function $r = r(x,t)$ satisfies the linear equation

$$\frac{\partial r}{\partial t} = a\frac{\partial^2 r}{\partial x^2} + rf(\tan\varphi) + h(\tan\varphi). \tag{1}$$

The change of variable

$$r = F(t)\left[Z(x,t) + \int\frac{h(\tan\varphi)\,dt}{F(t)}\right], \qquad F(t) = \exp\left[\int f(\tan\varphi)\,dt\right]$$

brings (1) to the linear heat equation

$$\frac{\partial Z}{\partial t} = a\frac{\partial^2 Z}{\partial x^2}.$$

