13. $$\frac{\partial u}{\partial t} = a\frac{\partial^2 u}{\partial x^2} + uf\Big(\frac{w}{u}\Big) + wg\Big(\frac{w}{u}\Big) + \frac{u}{\sqrt{u^2-w^2}}h\Big(\frac{w}{u}\Big),$$
$$\frac{\partial w}{\partial t} = a\frac{\partial^2 w}{\partial x^2} + wf\Big(\frac{w}{u}\Big) + ug\Big(\frac{w}{u}\Big) + \frac{w}{\sqrt{u^2-w^2}}h\Big(\frac{w}{u}\Big).$$

Solution:
$$u = r(x,t)\cosh\varphi(t), \qquad w = r(x,t)\sinh\varphi(t),$$
where the function $\varphi = \varphi(t)$ is determined by the separable first-order ordinary differential equation
$$\varphi'_t = g(\tanh\varphi),$$
and the function $r = r(x,t)$ satisfies the linear equation
$$\frac{\partial r}{\partial t} = a\frac{\partial^2 r}{\partial x^2} + rf(\tanh\varphi) + h(\tanh\varphi). \tag{1}$$
The change of variable
$$r = F(t)\Big[Z(x,t) + \int\frac{h(\tanh\varphi)\,dt}{F(t)}\Big], \qquad F(t) = \exp\Big[\int f(\tanh\varphi)\,dt\Big]$$
brings (1) to the linear heat equation
$$\frac{\partial Z}{\partial t} = a\frac{\partial^2 Z}{\partial x^2}.$$

