14. $\dfrac{\partial u}{\partial t} = a\dfrac{\partial^2 u}{\partial x^2} + uf(u^n w^m), \qquad \dfrac{\partial w}{\partial t} = b\dfrac{\partial^2 w}{\partial x^2} + wg(u^n w^m).$

Solution:

$$u = e^{m(kx-\lambda t)}y(\xi), \qquad w = e^{-n(kx-\lambda t)}z(\xi), \qquad \xi = \beta x - \gamma t,$$

where k, λ, β, and γ are arbitrary constants, and the functions $y = y(\xi)$ and $z = z(\xi)$ are determined by the system of ordinary differential equations

$$a\beta^2 y''_{\xi\xi} + (2akm\beta + \gamma)y'_\xi + m(ak^2m + \lambda)y + yf(y^n z^m) = 0,$$
$$b\beta^2 z''_{\xi\xi} + (-2bkn\beta + \gamma)z'_\xi + n(bk^2n - \lambda)z + zg(y^n z^m) = 0.$$

To the special case $k = \lambda = 0$ there corresponds a traveling-wave solution.

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