16. $\dfrac{\partial u}{\partial t} = a\dfrac{\partial^2 u}{\partial x^2} + cu\ln u + uf(u^n w^m), \qquad \dfrac{\partial w}{\partial t} = b\dfrac{\partial^2 w}{\partial x^2} + cw\ln w + wg(u^n w^m).$

Solution:

$$u = \exp(Ame^{ct})y(\xi), \quad w = \exp(-Ane^{ct})z(\xi), \quad \xi = kx - \lambda t,$$

where A, k, and λ are arbitrary constants, and the functions $y = y(\xi)$ and $z = z(\xi)$ are determined by the system of ordinary differential equations

$$ak^2 y''_{\xi\xi} + \lambda y'_{\xi} + cy\ln y + yf(y^n z^m) = 0,$$
$$bk^2 z''_{\xi\xi} + \lambda z'_{\xi} + cz\ln z + zg(y^n z^m) = 0.$$

To the special case $A = 0$ there corresponds a traveling-wave solution. If $\lambda = 0$, there exists a solution in the form of the product of two functions with arguments t and x, respectively.

Reference

Polyanin, A. D., Exact solutions of nonlinear systems of reaction-diffusion equations and mathematical biology equations, *Theor. Found. Chem. Eng.*, Vol. 37, No. 6, 2004.

