17. $\dfrac{\partial u}{\partial t}=a\dfrac{\partial^2 u}{\partial x^2}+uf(u^2+w^2)-wg(u^2+w^2),\qquad \dfrac{\partial w}{\partial t}=a\dfrac{\partial^2 w}{\partial x^2}+wf(u^2+w^2)+ug(u^2+w^2).$

1°. Periodic solution in the space coordinate with phase shift in components:

$$u=\psi(t)\cos\varphi(x,t),\qquad w=\psi(t)\sin\varphi(x,t),\qquad \varphi(x,t)=C_1x+\int g(\psi^2)\,dt+C_2,$$

where C_1 and C_2 are arbitrary constants, and the function $\psi=\psi(t)$ is determined by the separable first-order ordinary differential equation

$$\psi'_t=\psi f(\psi^2)-aC_1^2\psi,$$

whose general solution can be represented in implicit form as

$$\int\frac{d\psi}{\psi f(\psi^2)-aC_1^2\psi}=t+C_3.$$

2°. Periodic solution in time with phase shift in components:

$$u=r(x)\cos\bigl[\theta(x)+C_1t+C_2\bigr],\qquad w=r(x)\sin\bigl[\theta(x)+C_1t+C_2\bigr],$$

where C_1 and C_2 are arbitrary constants, and the functions $r=r(x)$ and $\theta=\theta(x)$ are determined by the system of ordinary differential equations

$$ar''_{xx}-ar(\theta'_x)^2+rf(r^2)=0,$$
$$ar\theta''_{xx}+2ar'_x\theta'_x-C_1r+rg(r^2)=0.$$

3°. Solution (generalizes the solution o Item 2°):

$$u=r(z)\cos\bigl[\theta(z)+C_1t+C_2\bigr],\qquad w=r(z)\sin\bigl[\theta(z)+C_1t+C_2\bigr],\qquad z=x+\lambda t,$$

where C_1, C_2, and λ are arbitrary constants, and the functions $r=r(z)$ and $\theta=\theta(z)$ are determined by the system of ordinary differential equations

$$ar''_{zz}-ar(\theta'_z)^2-\lambda r'_z+rf(r^2)=0,$$
$$ar\theta''_{zz}+2ar'_z\theta'_z-\lambda r\theta'_z-C_1r+rg(r^2)=0.$$

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