18. \[
\frac{\partial u}{\partial t} = a \frac{\partial^2 u}{\partial x^2} + uf(u^2 - w^2) + wg(u^2 - w^2), \quad \frac{\partial w}{\partial t} = a \frac{\partial^2 w}{\partial x^2} + wf(u^2 - w^2) + ug(u^2 - w^2).
\]

1°. Solution:

\[
u = \psi(t) \cosh \varphi(x, t), \quad w = \psi(t) \sinh \varphi(x, t), \quad \varphi(x, t) = C_1 x + \int g(\psi^2) \, dt + C_2,
\]
where \(C_1\) and \(C_2\) are arbitrary constants, and the function \(\psi = \psi(t)\) is determined by the separable first-order ordinary differential equation

\[
\psi' = \psi f(\psi^2) + aC_1 \psi,
\]
whose general solution can be represented in implicit form as

\[
\int \frac{d\psi}{\psi f(\psi^2) + aC_1 \psi} = t + C_3.
\]

2°. Solution:

\[
u = r(x) \cosh[\theta(x) + C_1 t + C_2], \quad w = r(x) \sinh[\theta(x) + C_1 t + C_2],
\]
where \(C_1\) and \(C_2\) are arbitrary constants, and the functions \(r = r(x)\) and \(\theta = \theta(x)\)

\[
ar''_x + ar(\theta'_x)^2 + rf(r^2) = 0, \quad ar'_{xx} + 2ar'_x \theta'_x + rg(r^2) - C_1 r = 0.
\]

3°. Solution (generalizes the solution of Item 2°):

\[
u = r(z) \cosh[\theta(z) + C_1 t + C_2], \quad w = r(z) \sinh[\theta(z) + C_1 t + C_2], \quad z = x + \lambda t,
\]
where \(C_1\), \(C_2\), and \(\lambda\) are arbitrary constants, and the functions \(r = r(z)\) and \(\theta = \theta(z)\) are determined by the system of ordinary differential equations

\[
ar''_{zz} + ar(\theta'_z)^2 - \lambda r'_z + rf(r^2) = 0, \quad ar''_{zz} + 2ar'_z \theta'_z - \lambda r''_z - C_1 r + rg(r^2) = 0.
\]

Reference