18. $\dfrac{\partial u}{\partial t} = a\dfrac{\partial^2 u}{\partial x^2} + uf(u^2-w^2) + wg(u^2-w^2), \qquad \dfrac{\partial w}{\partial t} = a\dfrac{\partial^2 w}{\partial x^2} + wf(u^2-w^2) + ug(u^2-w^2).$

1°. Solution:

$$u = \psi(t)\cosh\varphi(x,t), \qquad w = \psi(t)\sinh\varphi(x,t), \qquad \varphi(x,t) = C_1 x + \int g(\psi^2)\,dt + C_2,$$

where C_1 and C_2 are arbitrary constants, and the function $\psi = \psi(t)$ is determined by the separable first-order ordinary differential equation

$$\psi'_t = \psi f(\psi^2) + aC_1^2\psi,$$

whose general solution can be represented in implicit form as

$$\int \frac{d\psi}{\psi f(\psi^2) + aC_1^2\psi} = t + C_3.$$

2°. Solution:

$$u = r(x)\cosh\bigl[\theta(x) + C_1 t + C_2\bigr], \qquad w = r(x)\sinh\bigl[\theta(x) + C_1 t + C_2\bigr],$$

where C_1 and C_2 are arbitrary constants, and the functions $r = r(x)$ and $\theta = \theta(x)$

$$ar''_{xx} + ar(\theta'_x)^2 + rf(r^2) = 0,$$
$$ar\theta''_{xx} + 2ar'_x\theta'_x + rg(r^2) - C_1 r = 0.$$

3°. Solution (generalizes the solution of Item 2°):

$$u = r(z)\cosh\bigl[\theta(z) + C_1 t + C_2\bigr], \qquad w = r(z)\sinh\bigl[\theta(z) + C_1 t + C_2\bigr], \qquad z = x + \lambda t,$$

where C_1, C_2, and λ are arbitrary constants, and the functions $r = r(z)$ and $\theta = \theta(z)$ are determined by the system of ordinary differential equations

$$ar''_{zz} + ar(\theta'_z)^2 - \lambda r'_z + rf(r^2) = 0,$$
$$ar\theta''_{zz} + 2ar'_z\theta'_z - \lambda r\theta'_z - C_1 r + rg(r^2) = 0.$$

Reference

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