



19.
$$\frac{\partial u}{\partial t} = a \frac{\partial^2 u}{\partial x^2} + u f(u^2 + w^2) - w g(u^2 + w^2) - w \arctan\left(\frac{w}{u}\right) h(u^2 + w^2),$$
$$\frac{\partial w}{\partial t} = a \frac{\partial^2 w}{\partial x^2} + w f(u^2 + w^2) + u g(u^2 + w^2) + u \arctan\left(\frac{w}{u}\right) h(u^2 + w^2).$$

Functional separable solution (for fixed t , it defines a structure periodic in x with phase shift in components):

$$u = r(t) \cos[\varphi(t)x + \psi(t)], \quad w = r(t) \sin[\varphi(t)x + \psi(t)],$$

where the functions $r = r(t)$, $\varphi = \varphi(t)$, and $\psi = \psi(t)$ are determined by the system of ordinary differential equations

$$\begin{aligned} r'_t &= -ar\varphi^2 + rf(r^2), \\ \varphi'_t &= h(r^2)\varphi, \\ \psi'_t &= h(r^2)\psi + g(r^2). \end{aligned}$$

Reference

Polyanin, A. D., Exact solutions of nonlinear systems of reaction-diffusion equations and mathematical biology equations, *Theor. Found. Chem. Eng.*, Vol. 37, No. 6, 2004.