21. $\dfrac{\partial u}{\partial t} = a\dfrac{\partial^2 u}{\partial x^2} + u^{k+1} f(\varphi), \quad \dfrac{\partial w}{\partial t} = a\dfrac{\partial^2 w}{\partial x^2} + u^{k+1}[f(\varphi)\ln u + g(\varphi)], \quad \varphi = u\exp\left(-\dfrac{w}{u}\right).$

Solution:

$$u = (C_1 t + C_2)^{-\frac{1}{k}} y(\xi), \quad w = (C_1 t + C_2)^{-\frac{1}{k}} \left[z(\xi) - \frac{1}{k}\ln(C_1 t + C_2) y(\xi) \right], \quad \xi = \frac{x + C_3}{\sqrt{C_1 t + C_2}},$$

where C_1, C_2, and C_3 are arbitrary constants, and the functions $y = y(\xi)$ and $z = z(\xi)$ are determined by the system of ordinary differential equations

$$ay''_{\xi\xi} + \frac{1}{2}C_1 \xi y'_{\xi} + \frac{C_1}{k} y + y^{k+1} f(\varphi) = 0, \qquad \varphi = y\exp\left(-\frac{z}{y}\right),$$
$$az''_{\xi\xi} + \frac{1}{2}C_1 \xi z'_{\xi} + \frac{C_1}{k} z + \frac{C_1}{k} y + y^{k+1}[f(\varphi)\ln y + g(\varphi)] = 0.$$

Reference

Barannyk, T. A., Symmetry and exact solutions for systems of nonlinear reaction-diffusion equations, *Proc. of Inst. of Mathematics of NAS of Ukraine*, Vol. 43, Part 1, pp. 80–85, 2002.

