25. $\dfrac{\partial u}{\partial t}=a\dfrac{\partial^2 u}{\partial x^2}+uf\left(u^2-w^2,\dfrac{w}{u}\right)+wg\left(\dfrac{w}{u}\right),\qquad \dfrac{\partial w}{\partial t}=a\dfrac{\partial^2 w}{\partial x^2}+wf\left(u^2-w^2,\dfrac{w}{u}\right)+ug\left(\dfrac{w}{u}\right).$

Solution:

$$u=r(x,t)\cosh\varphi(t),\qquad w=r(x,t)\sinh\varphi(t),$$

where the function $\varphi=\varphi(t)$ is determined by the separable first-order ordinary differential equation

$$\varphi_t'=g(\tanh\varphi),$$

and the function $r=r(x,t)$ is determined by the differential equation

$$\frac{\partial r}{\partial t}=a\frac{\partial^2 r}{\partial x^2}+rf(r^2,\tanh\varphi).$$

