26. $\dfrac{\partial u}{\partial t} = a\dfrac{\partial^2 u}{\partial x^2} + F(u,w), \qquad \dfrac{\partial w}{\partial t} = a\dfrac{\partial^2 w}{\partial x^2} + bF(u,w).$

Let us multiply the first equation by $-b$ and add it to the second equation to obtain the linear heat equation

$$\frac{\partial \zeta}{\partial t} = a\frac{\partial^2 \zeta}{\partial x^2}, \qquad \zeta = w - bu, \tag{1}$$

which will be treated in conjunction with the first equation of the original system,

$$\frac{\partial u}{\partial t} = a\frac{\partial^2 u}{\partial x^2} + F(u, bu + \zeta). \tag{2}$$

For the simplest particular solution of equation (1),

$$\zeta = c,$$

where c is an arbitrary constant, equation (2) is a heat equation with a nonlinear source, whose exact solutions can be found in the "Handbook of Nonlinear Partial Differential Equations" by Polyanin & Zaitsev (2004).

