27. $\dfrac{\partial u}{\partial t} = a\dfrac{\partial^2 u}{\partial x^2} + uf(bu-cw) + g(bu-cw) + c\Phi(u,w),$

$$\frac{\partial w}{\partial t} = a\frac{\partial^2 w}{\partial x^2} + wf(bu-cw) + h(bu-cw) + b\Phi(u,w).$$

Let us multiply the first equation by b and add it to the second equation multiplied by $-c$ to obtain

$$\frac{\partial \zeta}{\partial t} = a\frac{\partial^2 \zeta}{\partial x^2} + \zeta f(\zeta) + bg(\zeta) - ch(\zeta), \qquad \zeta = bu - cw. \tag{1}$$

This equation will be treated in conjunction with the first equation of the original system,

$$\frac{\partial u}{\partial t} = a\frac{\partial^2 u}{\partial x^2} + uf(\zeta) + g(\zeta) + c\Phi\left(u, \frac{bu-\zeta}{c}\right). \tag{2}$$

Equation (1) can be treated separately. In the general case, it admits a space-homogeneous solution, $\zeta = \zeta(t)$, a stationary solution, $\zeta = \zeta(x)$, and a traveling-wave solution, $\zeta = \zeta(kx - \lambda t)$. An extensive list of exact solutions to equations of the form (1) for various kinetic functions $F(\zeta) = \zeta f(\zeta) + bg(\zeta) - ch(\zeta)$ can be found in the "Handbook of Nonlinear Partial Differential Equations" by Polyanin & Zaitsev (2004).

If the condition $\zeta f(\zeta) + bg(\zeta) - ch(\zeta) = 0$ holds, equation (1) is a linear heat equation, which admits particular solutions $\zeta = \zeta_0$ (ζ_0 is an arbitrary constant). In this case, equation (2) becomes a heat equation with nonlinear source, whose exact solutions can be found in the handbook cited above.

