3. \[ \frac{\partial u}{\partial t} = \frac{a}{x^n} \frac{\partial}{\partial x} \left( x^n \frac{\partial u}{\partial x} \right) + uf \left( \frac{u}{w} \right), \quad \frac{\partial w}{\partial t} = \frac{a}{x^n} \frac{\partial}{\partial x} \left( x^n \frac{\partial w}{\partial x} \right) + wg \left( \frac{u}{w} \right). \]

1°. Multiplicative separable solution:
\[ u = x^{1-n} \int \left[ C_1 J_{\nu}(kx) + C_2 Y_{\nu}(kx) \right] \varphi(t), \quad \nu = \frac{1}{2} \left| n - 1 \right|, \]
\[ w = x^{1-n} \int \left[ C_1 J_{\nu}(kx) + C_2 Y_{\nu}(kx) \right] \psi(t), \]
where \( C_1, C_2, \) and \( k \) are arbitrary constants, \( J_{\nu}(z) \) and \( Y_{\nu}(z) \) are the Bessel functions, and the functions \( \varphi = \varphi(t) \) and \( \psi = \psi(t) \) are determined by the system of ordinary differential equations
\[ \varphi' = -ak^2 \varphi + \varphi f(\varphi/\psi), \]
\[ \psi' = -ak^2 \psi + \psi g(\varphi/\psi). \]

2°. Multiplicative separable solution:
\[ u = x^{1-n} \int \left[ C_1 I_{\nu}(kx) + C_2 K_{\nu}(kx) \right] \varphi(t), \quad \nu = \frac{1}{2} \left| n - 1 \right|, \]
\[ w = x^{1-n} \int \left[ C_1 I_{\nu}(kx) + C_2 K_{\nu}(kx) \right] \psi(t), \]
where \( C_1, C_2, \) and \( k \) are arbitrary constants, \( I_{\nu}(z) \) and \( K_{\nu}(z) \) are the modified Bessel functions, and the functions \( \varphi = \varphi(t) \) and \( \psi = \psi(t) \) are determined by the system of ordinary differential equations
\[ \varphi' = ak^2 \varphi + \varphi f(\varphi/\psi), \]
\[ \psi' = ak^2 \psi + \psi g(\varphi/\psi). \]

3°. Multiplicative separable solution:
\[ u = e^{-\lambda t} y(x), \quad w = e^{-\lambda t} z(x), \]
where \( \lambda \) is an arbitrary constant and the functions \( y = y(x) \) and \( z = z(x) \) are determined by the system of ordinary differential equations
\[ ax^{-n}(x^n y')' + \lambda y + y f(y/z) = 0, \]
\[ ax^{-n}(x^n z')' + \lambda z + z g(y/z) = 0. \]

4°. Let \( k \) is a root of the algebraic (transcendental) equation
\[ f(k) = g(k). \]

Solution:
\[ u = ke^{\lambda t} \theta, \quad w = e^{\lambda t} \theta, \quad \lambda = f(k), \]
where the function \( \theta = \theta(x, t) \) satisfies the linear heat equation
\[ \frac{\partial \theta}{\partial t} = \frac{a}{x^n} \frac{\partial}{\partial x} \left( x^n \frac{\partial \theta}{\partial x} \right). \] (1)

5°. Solution:
\[ u = \varphi(t) \exp \left[ \int g(\varphi(t)) \, dt \right] \theta(x, t), \quad w = \exp \left[ \int g(\varphi(t)) \, dt \right] \theta(x, t), \]
where the function \( \varphi = \varphi(t) \) satisfies the separable nonlinear first-order ordinary differential equation
\[ \varphi' = \left[ f(\varphi) - g(\varphi) \right] \varphi, \] (2)
and the function \( \theta = \theta(x, t) \) satisfies the linear heat equation (1).

To the particular solution \( \varphi = k = \text{const} \) to equation (2) there corresponds the solution of Item 4°.

The general solution to equation (2) is written out in implicit form as
\[ \int \frac{d\varphi}{f(\varphi) - g(\varphi)} = t + C. \]