4. $\dfrac{\partial u}{\partial t} = \dfrac{a}{x^n}\dfrac{\partial}{\partial x}\left(x^n\dfrac{\partial u}{\partial x}\right) + uf\left(\dfrac{u}{w}\right), \quad \dfrac{\partial w}{\partial t} = \dfrac{b}{x^n}\dfrac{\partial}{\partial x}\left(x^n\dfrac{\partial w}{\partial x}\right) + wg\left(\dfrac{u}{w}\right).$

1°. Multiplicative separable solution:

$$u = x^{\frac{1-n}{2}}[C_1J_\nu(kx) + C_2Y_\nu(kx)]\varphi(t), \quad \nu = \tfrac{1}{2}|n-1|,$$
$$w = x^{\frac{1-n}{2}}[C_1J_\nu(kx) + C_2Y_\nu(kx)]\psi(t),$$

where C_1, C_2, and k are arbitrary constants, $J_\nu(z)$ and $Y_\nu(z)$ are the Bessel functions, and the functions $\varphi = \varphi(t)$ and $\psi = \psi(t)$ are determined by the system of ordinary differential equations

$$\varphi'_t = -ak^2\varphi + \varphi f(\varphi/\psi),$$
$$\psi'_t = -bk^2\psi + \psi g(\varphi/\psi).$$

2°. Multiplicative separable solution:

$$u = x^{\frac{1-n}{2}}[C_1I_\nu(kx) + C_2K_\nu(kx)]\varphi(t), \quad \nu = \tfrac{1}{2}|n-1|,$$
$$w = x^{\frac{1-n}{2}}[C_1I_\nu(kx) + C_2K_\nu(kx)]\psi(t),$$

where C_1, C_2, and k are arbitrary constants, $I_\nu(z)$ and $K_\nu(z)$ are the modified Bessel functions, and the functions $\varphi = \varphi(t)$ and $\psi = \psi(t)$ are determined by the system of ordinary differential equations

$$\varphi'_t = ak^2\varphi + \varphi f(\varphi/\psi),$$
$$\psi'_t = bk^2\psi + \psi g(\varphi/\psi).$$

3°. Multiplicative separable solution:

$$u = e^{-\lambda t}y(x), \quad w = e^{-\lambda t}z(x),$$

where λ is an arbitrary constant and the functions $y = y(x)$ and $z = z(x)$ are determined by the system of ordinary differential equations

$$ax^{-n}(x^ny'_x)'_x + \lambda y + yf(y/z) = 0,$$
$$bx^{-n}(x^nz'_x)'_x + \lambda z + zg(y/z) = 0.$$

