5. $$\frac{\partial u}{\partial t} = \frac{a}{x^n}\frac{\partial}{\partial x}\left(x^n\frac{\partial u}{\partial x}\right) + uf\left(\frac{u}{w}\right) + \frac{u}{w}h\left(\frac{u}{w}\right),$$
$$\frac{\partial w}{\partial t} = \frac{a}{x^n}\frac{\partial}{\partial x}\left(x^n\frac{\partial u}{\partial w}\right) + wg\left(\frac{u}{w}\right) + h\left(\frac{u}{w}\right).$$

Solution:

$$u = \varphi(t)G(t)\left[\theta(x,t) + \int\frac{h(\varphi)}{G(t)}\,dt\right], \quad w = G(t)\left[\theta(x,t) + \int\frac{h(\varphi)}{G(t)}\,dt\right], \quad G(t) = \exp\left[\int g(\varphi)\,dt\right],$$

where the function $\varphi = \varphi(t)$ is determined by the separable nonlinear first-order ordinary differential equation

$$\varphi'_t = [f(\varphi) - g(\varphi)]\varphi, \qquad (1)$$

and the function $\theta = \theta(x,t)$ satisfies the linear heat equation

$$\frac{\partial\theta}{\partial t} = \frac{a}{x^n}\frac{\partial}{\partial x}\left(x^n\frac{\partial\theta}{\partial x}\right).$$

The general solution of equation (1) is written out in implicit form as

$$\int\frac{d\varphi}{[f(\varphi) - g(\varphi)]\varphi} = t + C.$$

