**7.** $$\frac{\partial u}{\partial t} = \frac{a}{x^n}\frac{\partial}{\partial x}\left(x^n\frac{\partial u}{\partial x}\right) + uf\left(\frac{u}{w}\right)\ln u + ug\left(\frac{u}{w}\right),$$
$$\frac{\partial w}{\partial t} = \frac{a}{x^n}\frac{\partial}{\partial x}\left(x^n\frac{\partial w}{\partial x}\right) + wf\left(\frac{u}{w}\right)\ln w + wh\left(\frac{u}{w}\right).$$

Solution:

$$u = \varphi(t)\psi(t)\theta(x,t), \quad w = \psi(t)\theta(x,t),$$

where the functions $\varphi = \varphi(t)$ and $\psi = \psi(t)$ are determined by solving the ordinary differential equations

$$\begin{aligned} \varphi'_t &= \varphi[g(\varphi) - h(\varphi) + f(\varphi)\ln\varphi], \\ \psi'_t &= \psi[h(\varphi) + f(\varphi)\ln\psi], \end{aligned} \tag{1}$$

and the function $\theta = \theta(x,t)$ is determined by the differential equation

$$\frac{\partial\theta}{\partial t} = \frac{a}{x^n}\frac{\partial}{\partial x}\left(x^n\frac{\partial\theta}{\partial x}\right) + f(\varphi)\theta\ln\theta. \tag{2}$$

The first equation in (1) is separable; its solution can be written out in implicit form. The second equation in (1) can be solved using the change of variable $\psi = e^{\zeta}$ (it is reduced to a linear equation for $\zeta$).

Equation (2) admits exact solutions of the form

$$\theta = \exp\left[\sigma_2(t)x^2 + \sigma_0(t)\right],$$

where the functions $\sigma_n(t)$ are determined by the equations

$$\begin{aligned} \sigma'_2 &= f(\varphi)\sigma_2 + 4a\sigma_2^2, \\ \sigma'_0 &= f(\varphi)\sigma_0 + 2a(n+1)\sigma_2. \end{aligned}$$

This system can be integrated successively, since the first equation is a Bernoulli equation and the second one is linear in the unknown.

If $f = \text{const}$, equation (2) has also a traveling-wave solution $\theta = \theta(kx - \lambda t)$.

 