



$$11. \quad \frac{\partial u}{\partial t} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial u}{\partial x} \right) + u f(u^2 + w^2) - w g(u^2 + w^2),$$
$$\frac{\partial w}{\partial t} = \frac{a}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial w}{\partial x} \right) + w f(u^2 + w^2) + u g(u^2 + w^2).$$

Periodic (in time) solution:

$$u = r(x) \cos[\theta(x) + C_1 t + C_2], \quad w = r(x) \sin[\theta(x) + C_1 t + C_2],$$

where C_1 and C_2 are arbitrary constants, and the function $r = r(x)$ are determined by the system of ordinary differential equations

$$ar''_{xx} - ar(\theta'_x)^2 + \frac{an}{x} r'_x + r f(r^2) = 0,$$
$$ar\theta''_{xx} + 2ar'_x \theta'_x + \frac{an}{x} r\theta'_x + r g(r^2) - C_1 r = 0.$$