12. $\dfrac{\partial u}{\partial t} = \dfrac{a}{x^n}\dfrac{\partial}{\partial x}\left(x^n \dfrac{\partial u}{\partial x}\right) + uf(u^2 - w^2) + wg(u^2 - w^2),$

$$\frac{\partial w}{\partial t} = \frac{a}{x^n}\frac{\partial}{\partial x}\left(x^n \frac{\partial w}{\partial x}\right) + wf(u^2 - w^2) + ug(u^2 - w^2).$$

Solution:

$$u = r(x)\cosh\bigl[\theta(x) + C_1 t + C_2\bigr], \qquad w = r(x)\sinh\bigl[\theta(x) + C_1 t + C_2\bigr],$$

where C_1 and C_2 are arbitrary constants, and the function $r = r(x)$ are determined by the system of ordinary differential equations

$$ar''_{xx} + ar(\theta'_x)^2 + \frac{an}{x}r'_x + rf(r^2) = 0,$$
$$ar\theta''_{xx} + 2ar'_x\theta'_x + \frac{an}{x}r\theta'_x + rg(r^2) - C_1 r = 0.$$

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