13. $$\frac{\partial u}{\partial t} = \frac{a}{x^n}\frac{\partial}{\partial x}\left(x^n\frac{\partial u}{\partial x}\right) + uf(u^2+w^2) - wg\left(\frac{w}{u}\right),$$
$$\frac{\partial w}{\partial t} = \frac{a}{x^n}\frac{\partial}{\partial x}\left(x^n\frac{\partial w}{\partial x}\right) + wf(u^2+w^2) + ug\left(\frac{w}{u}\right).$$

Solution:
$$u = r(x,t)\cos\varphi(t), \quad w = r(x,t)\sin\varphi(t),$$
where the function $\varphi = \varphi(t)$ is determined by the autonomous ordinary differential equation
$$\varphi'_t = g(\tan\varphi), \tag{1}$$
and the function $r = r(x,t)$ is determined by the differential equation
$$\frac{\partial r}{\partial t} = \frac{a}{x^n}\frac{\partial}{\partial x}\left(x^n\frac{\partial r}{\partial x}\right) + rf(r^2). \tag{2}$$

The general solution of equation (1) is expressed in implicit form as
$$\int \frac{d\varphi}{g(\tan\varphi)} = t + C.$$
Equation (2) admits a stationary exact solution $r = r(x)$.

Remark. The function f can also be explicitly dependent on x.

