14. $\dfrac{\partial u}{\partial t} = \dfrac{a}{x^n}\dfrac{\partial}{\partial x}\Big(x^n\dfrac{\partial u}{\partial x}\Big) + uf(u^2-w^2) + wg\Big(\dfrac{w}{u}\Big),$

$$\frac{\partial w}{\partial t} = \frac{a}{x^n}\frac{\partial}{\partial x}\Big(x^n\frac{\partial w}{\partial x}\Big) + wf(u^2-w^2) + ug\Big(\frac{w}{u}\Big).$$

Solution:

$$u = r(x,t)\cosh\varphi(t), \quad w = r(x,t)\sinh\varphi(t),$$

where the function $\varphi = \varphi(t)$ is determined by the autonomous ordinary differential equation

$$\varphi'_t = g(\tanh\varphi), \tag{1}$$

and the function $r = r(x,t)$ is determined by the differential equation

$$\frac{\partial r}{\partial t} = \frac{a}{x^n}\frac{\partial}{\partial x}\Big(x^n\frac{\partial r}{\partial x}\Big) + rf(r^2). \tag{2}$$

The general solution of equation (1) is expressed in implicit form as

$$\int\frac{d\varphi}{g(\tanh\varphi)} = t + C.$$

Equation (2) admits a stationary exact solution $r = r(x)$.

Remark. The function f can also be explicitly dependent on x.

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