1. \[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = u f(a u - b w) + g(a u - b w), \quad \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = w f(a u - b w) + h(a u - b w). \]

1°. Solution:

\[ u = \varphi(x) + b \theta(x, y), \quad w = \psi(x) + a \theta(x, y), \]

where \( \varphi = \varphi(x) \) and \( \psi = \psi(x) \) are determined by the system of ordinary differential equations

\[ \varphi''_{xx} = \varphi f(a \varphi - b \psi) + g(a \varphi - b \psi), \quad \psi''_{xx} = \psi f(a \varphi - b \psi) + h(a \varphi - b \psi), \]

and the function \( \theta = \theta(x, y) \) satisfies a linear Schrödinger equation of the special form

\[ \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = F(x) \theta, \quad F(x) = f(a u - b w). \]

Its solutions are constructed by the method of separation of variables.

2°. Let us multiply the first equation by \( a \) and add it to the second equation multiplied by \( -b \) to obtain

\[ \frac{\partial^2 \zeta}{\partial x^2} + \frac{\partial^2 \zeta}{\partial y^2} = \zeta f(\zeta) + ag(\zeta) - bh(\zeta), \quad \zeta = au - bw. \quad (1) \]

This equation will be treated in conjunction with the first equation of the original system

\[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = uf(\zeta) + g(\zeta). \quad (2) \]

Equation (1) can be treated separately. An extensive list of exact solutions to this sort of equations for various kinetic functions \( F(\zeta) = \zeta f(\zeta) + ag(\zeta) - bh(\zeta) \) can be found in the “Handbook of Nonlinear Partial Differential Equations” by A. D. Polyanin & V. F. Zaitsev (2004).

Note two important cases:

(i) In the general case equation (1) admits a traveling-wave solution \( \zeta = \zeta(z) \), where \( z = k_1 x + k_2 y \) (\( k_1 \) and \( k_2 \) are arbitrary constants).

(ii) If the condition \( \zeta f(\zeta) + ag(\zeta) - bh(\zeta) = c_1 \zeta + c_0 \) holds, equation (1) is a linear Helmholtz equation.

Given a solution \( \zeta = \zeta(x, y) \) of equation (1), the function \( u = u(x, y) \) can be found by solving the linear equation (2), and the function \( w = w(x, y) \) is determined by the formula \( w = (bu - \zeta)/c. \)