2. $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}=e^{\lambda u}f(\lambda u-\sigma w),\quad \dfrac{\partial^2 w}{\partial x^2}+\dfrac{\partial^2 w}{\partial y^2}=e^{\sigma w}g(\lambda u-\sigma w).$

1°. Solution:

$$u=U(\xi)-\frac{2}{\lambda}\ln|x+C_1|,\qquad w=W(\xi)-\frac{2}{\sigma}\ln|x+C_1|,\qquad \xi=\frac{y+C_2}{x+C_1},$$

where C_1 and C_2 are arbitrary constants, and the functions $U=U(\xi)$ and $W=W(\xi)$ are determined by the system of ordinary differential equations

$$(1+\xi^2)U''_{\xi\xi}+2\xi U'_{\xi}+\frac{2}{\lambda}=e^{\lambda U}f(\lambda U-\sigma W),$$
$$(1+\xi^2)W''_{\xi\xi}+2\xi W'_{\xi}+\frac{2}{\sigma}=e^{\sigma W}g(\lambda U-\sigma W).$$

2°. Solution:

$$u=\theta(x,y),\qquad w=\frac{\lambda}{\sigma}\theta(x,y)-\frac{k}{\sigma},$$

where k is a root of the algebraic (transcendental) equation

$$\lambda f(k)=\sigma e^{-k}g(k),$$

and the function $\theta=\theta(x,y)$ satisfies the solvable equation

$$\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}=f(k)e^{\lambda\theta}.$$

This equation is encountered in combustion theory. For its exact solutions, see the "Handbook of Nonlinear Partial Differential Equations" by A. D. Polyanin & V. F. Zaitsev (2004).

