5. $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = u^n f\Bigl(\dfrac{u}{w}\Bigr), \quad \dfrac{\partial^2 w}{\partial x^2} + \dfrac{\partial^2 w}{\partial y^2} = w^n g\Bigl(\dfrac{u}{w}\Bigr).$

If $f(z) = kz^{-m}$ and $g(z) = -kz^{n-m}$, the system in question describes a chemical reaction of order n (order $n-m$ in the u-component and order m in the w-component); to the values $n = 2$, $m = 1$ there corresponds a fairly common reaction of the second order.

1°. Solution:

$$u = r^{\frac{2}{1-n}} U(\theta), \quad w = r^{\frac{2}{1-n}} W(\theta), \quad r = \sqrt{(x + C_1)^2 + (y + C_2)^2}, \quad \theta = \frac{y + C_2}{x + C_1},$$

where C_1 and C_2 are arbitrary constants, and the functions $y = y(\xi)$ and $z = z(\xi)$ are determined by the autonomous system of ordinary differential equations

$$U''_{\theta\theta} + \frac{4}{(1-n)^2} U = U^n f\Bigl(\frac{U}{W}\Bigr),$$
$$W''_{\theta\theta} + \frac{4}{(1-n)^2} W = W^n g\Bigl(\frac{U}{W}\Bigr).$$

2°. Solution:

$$u = k\zeta(x, y), \quad w = \zeta(x, y),$$

where k is a root of the algebraic (transcendental) equation

$$k^{n-1} f(k) = g(k),$$

and the function $\zeta = \zeta(x, y)$ satisfies the equation with power-law nonlinearity

$$\frac{\partial^2 \zeta}{\partial x^2} + \frac{\partial^2 \zeta}{\partial y^2} = g(k)\zeta^n.$$